



No. 2003–61

**ENVIRONMENTAL POLICY, THE PORTER HYPOTHESIS
AND THE COMPOSITION OF CAPITAL: EFFECTS
OF LEARNING AND TECHNOLOGICAL PROGRESS**

By G. Feichtinger, R.F. Hartl, P.M. Kort,
V.M. Veliov

June 2003

ISSN 0924-7815

Environmental Policy, the Porter Hypothesis and the Composition of Capital: Effects of Learning and Technological Progress¹

Gustav Feichtinger

Institute for Econometrics, Operations Research and Systems Theory,
Vienna University of Technology, Vienna, Austria

Richard F. Hartl

Institute of Management Science, University of Vienna, Vienna, Austria

Peter M. Kort

Department of Econometrics & Operations Research and CentER,
Tilburg University, Tilburg, The Netherlands
Department of Economics, UFSIA University of Antwerp, Antwerp, Belgium

Vladimir M. Veliov

Institute for Econometrics, Operations Research and Systems Theory,
Vienna University of Technology, Vienna, Austria
Institute of Mathematics and Informatics, Bulgarian Acad. of Sciences, Sofia, Bulgaria

¹This research was partly supported by the Austrian Science Foundation under contract N0. 15618-OEK

Environmental Policy, the Porter Hypothesis and the Composition of Capital: Effects of Learning and Technological Progress

Abstract

In this paper the effect of environmental policy on the composition of capital is investigated. By allowing for non-linearities it generalizes Xepapadeas and De Zeeuw (Journal of Environmental Economics and Management, 1999) and determines scenarios in which their results do not carry over. In particular, we show that the way acquisition cost of investment decreases with the age of the capital stock is of crucial importance. Also it is obtained that environmental policy has opposite effects on the average age of the capital stock in the case of either deterioration or depreciation. We also focus more explicitly on learning and technological progress. Among others we obtain that in the presence of learning, implementing a stricter environmental policy with the aim to reach a certain target of emissions reduction has a stronger negative effect on industry profits, which implies quite the opposite as to what is described by the Porter hypothesis.

Keywords: Porter hypothesis, dynamics of the firm, vintage capital stock, environmental policy, technological progress

JEL-codes: C61, O33, Q28

1 Introduction

The Porter hypothesis says that environmental policy spurs innovation which makes firms better off in the long run, since it increases their competitiveness (Porter [13], Porter and van der Linde [14]). The main argument is that firms are not aware of certain opportunities and that environmental policy might open the eyes. This results in a win-win situation in the sense that environmental policy improves both environment and competitiveness. The hypothesis is criticized by economists who argue that extra costs are not needed to trigger fruitful innovations and adopting modern machines that are more profitable. In rational economic modelling it cannot be explained why firms do not see these opportunities by themselves, which at least implies that the argument does not have a general validity.

The validity of the Porter hypothesis was investigated in a very interesting contribution by Xepapadeas and de Zeeuw [16] in the context of a dynamic model of the firm. In this model firms can invest in machines of different ages (see also Barucci and Gozzi [1], Hartl et al. [11], and Feichtinger et al. [10]²) so that it could be investigated in what way environmental policy modernizes the machine park. It was found that more stringent environmental policy reduces capital stock (*downsizing*), and reduces the average age of the machines (*modernization*). Newer machines are assumed to be more productive, so that the modernization effect results in an increase of average productivity. The conclusion concerning the validity of the Porter hypothesis is that a win-win situation will not hold, but the decrease in competitiveness due to the extra environmental costs is mitigated by the modernization effect.

The economic reason for modernization was that modern machines, although being more expensive, are more productive and pollute less. This can be caused by natural deterioration due to aging, or technological progress. In Xepapadeas and De Zeeuw [16] only the effect of aging was taken into account. Although they say otherwise, we actually will show that their model does not contain technological progress. This implies that the effect of technological progress remains to be investigated, which will be done in the present paper.

Xepapadeas and De Zeeuw obtained this result under linear specifications while ignoring depreciation. In this paper we generalize their framework by allowing for non-linear functional forms and technological progress. It was found that the result of Xepapadeas and De Zeeuw is still valid as long as acquisition costs of investment are concavely decreasing in age. However, in case this function is convexly shaped average productivity of the capital stock can decrease when a stricter environmental policy is imposed.

Another interesting finding is that environmental policy has opposite effects on the average age of the capital stock in the case of either deterioration or depreciation. In the case of depreciation, but without deterioration, the average age of the capital stock increases when the emission tax is raised, while with deterioration it is the other way round.

As already mentioned, in Xepapadeas and De Zeeuw [16] productivity goes down and pollution increases as machines get older. However, due to learning the opposite is also possible. We investigate how learning affects the results obtained by Xepapadeas and De Zeeuw [16]. As a result we found that in the presence of learning, under a stricter environmental policy downsizing is not necessarily accompanied by modernization but could even be accompanied by a raise of the average

²These contributions (including the present paper) are different from works like Solow et al. [15], Malcomson [12], Benhabib and Rustichini [2], and Boucekkin et al. ([3], [4], [5], [6]) in the sense that also investments in older machines are possible. Economic reasons for doing so are that older machines are cheaper, easier to implement, and learning can make them more productive and less polluting.

age of capital stock. The reason is that when the emission tax rate is increased, firms react by increasing the production per unit of emissions. In the presence of natural deterioration (Xepapadeas and De Zeeuw) the increased productivity is reached by modernization, while in the presence of learning (our model) productivity per unit of emissions is increased by employing older machines. But again it has to be remarked that the shape of the acquisition cost function is crucial: this time the above results may not hold in case of a concave function.

Furthermore it was found that learning reduces the effect of a stricter environmental policy on emissions. This result leads to a serious policy warning that is especially valid for situations where learning is prominently present. It holds that, in comparison with a situation where learning is not present, an emission tax rate has to be set at a higher rate in order to reach a given emission reduction target, and this leads to a larger negative effect on the firm's profits.

Xepapadeas and De Zeeuw [16] limit their analysis to the OSSP (Optimal Steady State Problem, see Carlson et al. [7]) without actually verifying whether the optimal solution will reach this steady state. We will show that it is in fact optimal for the firm to reach this steady state as soon as possible, which strengthens the validity of their results.

The paper is organized as follows. Section 2 reformulates the model of Xepapadeas and De Zeeuw [16] where we add depreciation and discounting. Here we show that the firm always converges to the steady state. In Section 3 we study the effects of nonlinearities, and of deterioration and depreciation of capital on the emission tax response of the average age and productivity of the machines. In Section 4 the implications of learning are investigated. Technological progress is considered in Section 5, while Section 6 concludes. All proofs and technical considerations are collected in the Appendix, where also the precise assumptions are formulated in mathematical terms.

2 The Model

Here we present a slightly extended version of the model of Xepapadeas and De Zeeuw [16] in the sense that discounting and depreciation are added. As in their paper, here it also holds that the age of the machine is denoted by $a \in [0, h]$, so that the maximum age of the machines is h . As usual, calendar time is denoted by t .

Denote by $v(a)$ the output produced by a machine of age a , with $v'(a) \leq 0$. That is, an older machine cannot produce more output than a newer one. This model feature is taken from Xepapadeas and De Zeeuw [16] who argue that this implies that new machines are more productive because they embody superior technology. However, this argument seems to be incorrect. To see this, note that $v(a)$ is the same for different t . Now consider two points of time: t_1 and t_2 so that $t_2 > t_1$. Then a machine constructed at time t_2 has the same productivity at the same age as a machine constructed at time t_1 , i.e. the second machine produces at $t_2 + a$ amount $v(a)$, which is also the amount that the first machine produces at $t_1 + a$. Hence there is no superior technology embedded in the machine constructed at t_2 . The conclusion is that, since v is independent of calendar time t , no technological progress is included. In order to include technological progress, output should be modelled by $v(t, a)$ with, at least, $v_t > 0$ (see Hartl et al. [11] and Feichtinger et al. [10]). We will explore this further in Section 5. In this and the next two sections we only consider the case of time-invariant v .

In fact, $v'(a) \leq 0$ can still be a sensible assumption, because due to natural deterioration or aging machines can get less productive as time passes (see also Feichtinger et al. [9]). However,

the opposite, i.e. $v'(a) > 0$, is also possible since this reflects that due to learning machines get more productive over the years (cf. Hartl et al. [11]). The effects of environmental regulation in the presence of learning will be analyzed in Section 4.

The stock of capital goods of age a at time t is denoted by $K(t, a)$. Then total output produced in year t is defined as

$$Q(t) = \int_0^h v(a)K(t, a)da.$$

It is assumed that markets exist for machines of any age from 0 to h . This is a strong assumption but it is somewhat relaxed later on by the introduction of adjustment costs which may contain search costs. Let $b(a)$ be the cost of buying a machine of age a , with $b'(a) \leq 0$ (older machines cannot be more expensive than newer machines) and $b(h) = 0$ (a machine at the maximum age is not worth anything).

Let $I(t, a)$ be the number of machines of age a bought (if $I(t, a) > 0$) or sold (if $I(t, a) < 0$) in year t . The total cost or revenue to the firm from transactions in the machine market is defined as $b(a)I(t, a) + \frac{1}{2} [I(t, a)]^2$, with the second term reflecting the adjustment costs in buying or selling machines. These costs are, for example, adaptation costs or search costs. The quadratic form of this cost term leads to a simple expression for optimal purchases. In addition to Xepapadeas and De Zeeuw [16] it is further imposed that machines of age a depreciate with rate $\delta(a)$, which is the same for every vintage.

Running the machine is costly, and the running costs of a machine of age a are denoted by $c(a)$, $c'(a) \geq 0$.

Furthermore, $s(a)$ are emissions of a machine of age a , $s'(a) \geq 0$. As a result of natural deterioration, older machines emit at least as much as newer machines (but, analogous to our argument in connection with $v(a)$, not as a “result of cleaner technologies being embodied in the new machines” (Xepapadeas and De Zeeuw [16], p. 168). Here also learning effects may be present resulting in $s' \leq 0$: over time firms learn how to use machines in a more environmental friendly way. We will consider this in Section 4.

Per unit of emissions the firm has to pay an emission tax τ . Then the cost of running one machine is $c(a) + \tau s(a)$, which results in total running costs for year t being equal to

$$C(t) = \int_0^h [c(a) + \tau s(a)] K(t, a) da.$$

The firm chooses to buy or sell machines of different ages in order to maximize profits, with p the price of output. That is, the firm chooses at each point in time an age distribution of machines to maximize profits. Discounting the future with a rate $r \geq 0$, the dynamic model of the firm is now given by

$$\max_{I(t, a)} \int_0^\infty e^{-rt} \int_0^h [pv(a)K(t, a) - [c(a) + \tau s(a)] K(t, a)] da dt \quad (1)$$

$$- \int_0^\infty e^{-rt} \int_0^h \left[b(a)I(t, a) + \frac{1}{2} [I(t, a)]^2 \right] da dt$$

$$\text{subject to } \frac{\partial K(t, a)}{\partial t} + \frac{\partial K(t, a)}{\partial a} = I(t, a) - \delta(a) K(t, a), \quad (2)$$

$$K(t, 0) = I_0(t) \geq 0, \quad K(0, a) = K_0(a) \geq 0. \quad (3)$$

$$K(t, a) \geq 0, \quad (4)$$

This is an infinite horizon optimal control problem with transition dynamics described by a linear partial differential equation (Carlson, Haurie and Leizarowitz [7])³. The transition equation indicates that the rate of change in the number of machines of a given age, a , at a given time, t , is determined by two factors. These are the reduction or increase in the number of machines brought about by the sale or acquisition of machines of given age a (the first term of the transition equation), and the reduction due to depreciation at rate $\delta(a)$. The investment $I(t, a)$ is considered as a control variable. The initial condition on the number of machines implies that the firm starts with given amount $K_0(a)$ of machines of age a . At each time t it is possible to buy new machines. The purchase intensity of new machines is denoted by I_0 . Both $K_0(a)$ and $I_0(t)$ can also be considered as (initial and boundary) control variables, in which case the corresponding costs should be added to the objective function (1). However, we do not expect this to bring new economic results. Therefore, in what follows we consider $K_0(a)$ and $I_0(t)$ as given, and take for simplicity $I_0(t) = 0$.

The current value Hamiltonian H for this problem is given by (see, e.g., Feichtinger and Hartl [8]):

$$\begin{aligned} H = & \quad pv(a)K(t, a) - [c(a) + \tau s(a)] K(t, a) - b(a)I(t, a) - \frac{1}{2} [I(t, a)]^2 \\ & + \lambda(t, a) [I(t, a) - \delta(a) K(t, a)]. \end{aligned} \quad (5)$$

Consequently, the first-order optimality conditions are

$$\frac{\partial H}{\partial I} = 0, \quad \text{or} \quad I(t, a) = \lambda(t, a) - b(a), \quad (6)$$

$$\frac{\partial \lambda(t, a)}{\partial t} + \frac{\partial \lambda(t, a)}{\partial a} = r\lambda - \frac{\partial H}{\partial K} = (r + \delta(a)) \lambda(t, a) - pv(a) + c(a) + \tau s(a), \quad (7)$$

$$\lambda(t, h) = 0. \quad (8)$$

Solving the partial differential equation (7), while taking into account the boundary condition (8) yields:

$$\lambda(t, a) = \int_a^h e^{-\int_a^\alpha (r+\delta(\rho))d\rho} [pv(\alpha) - c(\alpha) - \tau s(\alpha)] d\alpha. \quad (9)$$

From (6) and (9) the optimal investment rate is established:

$$I(t, a) = \int_a^h e^{-\int_a^\alpha (r+\delta(\rho))d\rho} [pv(\alpha) - c(\alpha) - \tau s(\alpha)] d\alpha - b(a). \quad (10)$$

An expression for the stock of capital goods can be derived from (2), assuming for the moment that $a \leq t$:

$$K(t, a) = \left(\int_0^a e^{\int_0^\sigma \delta(\rho)d\rho} I(t + \sigma - a, \sigma) d\sigma \right) e^{-\int_0^a \delta(\rho)d\rho}. \quad (11)$$

³About the precise meaning of optimality, see Remark 1 in Section 7.1

Combining the last three expressions, we obtain for $t \geq a$ that

$$K(t, a) = \left(\int_0^a e^{\int_0^\sigma \delta(\rho) d\rho} \left[\int_\sigma^h e^{-\int_\sigma^\alpha (r+\delta(\rho)) d\rho} [pv(\alpha) - c(\alpha) - \tau s(\alpha)] d\alpha - b(\sigma) \right] d\sigma \right) e^{-\int_0^a \delta(\rho) d\rho}. \quad (12)$$

In case $t < a$, i.e. the vintage already exists at the initial time, it is easily derived via the second boundary condition in (3) that

$$K(t, a) = K_0(a-t) e^{-\int_{a-t}^a \delta(\rho) d\rho} + \int_{a-t}^a e^{-\int_\sigma^a \delta(\rho) d\rho} \left[\int_\sigma^h e^{-\int_\sigma^\alpha (r+\delta(\rho)) d\rho} [pv(\alpha) - c(\alpha) - \tau s(\alpha)] d\alpha - b(\sigma) \right] d\sigma. \quad (13)$$

An implicit assumption for the subsequent analysis is that the values of $K(t, a)$ obtained by the above formulas are all nonnegative, so that the constraint (4) is automatically satisfied. We elaborate on this in Section 7.1.

An important observation is that (9) and (10) are time invariant. Moreover, $K(t, a)$ depends on t only in case $t < a$. This means that after h years everything becomes time invariant, that is, the steady state with respect to calendar time is reached. We thus can state the following proposition.

Proposition 1 *At least after h years the firm reaches its steady state. For each age $a \in [0, h]$ the capital stock is then time invariant and given by (12). The investment rate in a capital stock of a given age is also constant over time and given by (10).*

3 Effects of Nonlinearities, Capital Deterioration and Depreciation on Modernization and Productivity

In this section we study whether introducing nonlinearities influences the impact of the emission tax on the average age and productivity of the capital stock. Also, we investigate the effects of the deterioration rate and the depreciation rate.

Average age of the optimal capital stock is defined by

$$g(\tau) = \frac{\int_0^h a K(t, a) da}{\int_0^h K(t, a) da},$$

while average productivity of the capital stock is given by

$$\pi(\tau) = \frac{\int_0^h v(a) K(t, a) da}{\int_0^h K(t, a) da}. \quad (14)$$

Due to the fact that they completely focus on natural deterioration and that they assumed a simple linear specification for $v(a)$, i.e.

$$v(a) = a_0 + a_1(h-a), \quad (15)$$

where a_0 is nonnegative and a_1 is strictly positive, Xepapadeas and De Zeeuw [16] obtained a monotonic relationship between modernization and productivity in the sense that the lower the

average age of the machines the higher the productivity is. Then they prove for linear specifications of $c(a)$, $b(a)$, and $s(a)$ that an increase in the emission tax reduces the optimal average age of the capital stock and increases its average productivity.

The following proposition extends this result to nonlinear functional forms.

Proposition 2 *Consider the model set up for the scenario without depreciation and discounting, thus $r = \delta(a) = 0$, where $v(a)$ is not identically zero and non-increasing in a , $c(a)$ and $s(a)$ are non-decreasing, and $b(a)$ is concave. Then at any time $t \geq h$ it holds that an increase in the emission tax rate*

i) reduces or keeps the optimal average age of the capital stock at the same level. If either $v(a)$ is strictly decreasing, or one of $c(a)$ and $s(a)$ is strictly increasing, or $b(a)$ is strictly concave, then the optimal average age strictly decreases with the increase of the tax rate;

ii) raises or keeps the average productivity of the capital stock at the same level. If $v(a)$ is strictly decreasing then the average productivity strictly increases with the increase of the tax rate.

The shape of the acquisition cost $b(a)$ turns out to be one of the essential determinants of the optimal average age and the average productivity of the capital stock. To focus on its implications for average age let us for the moment assume that all other functions are age independent, i.e.

$$v(a) = v, c(a) = c, s(a) = s.$$

Note that under this specification average productivity is constant and equal to v . Then it can be shown that for a linear specification of $b(a)$, e.g. $b_0(a) = b \cdot (h - a)$, which is the specification that Xepapadeas and De Zeeuw used to derive their result, it holds that average age is not influenced by an increase of the emission tax rate⁴.

In case of a strictly concave specification like, e.g.

$$b_2(a) = [h - a][b + a],$$

an increase of the emission tax rate reduces average age, as described in Proposition 2.

Next, consider the strictly convex function

$$b_1(a) = [h - a][b - a]$$

with $b \geq h$. The consideration in Section 7.1 used in the proof of Proposition 2 shows that now an increase of the tax rate leads to a higher average age. The conclusion is that the result obtained under a linear specification is not robust: an arbitrarily small nonlinear perturbation of the linear case leads to qualitatively different behavior. However, if the function $b(a)$ is strictly concave or

⁴In fact one can prove that average age and average productivity are also not influenced by an increase of the emission tax rate if $b(a)$ satisfies

$$\int_0^h (h^2 - 5h\sigma + 4\sigma^2)b(\sigma) d\sigma = 0. \quad (16)$$

The benchmark function $b_0(\cdot) = b \cdot (h - a)$ satisfies this equation. But it has also nonlinear solutions. Among the polynomials of degree not bigger than 2, the only function satisfying (16) and $b(h) = 0$ (modulo a constant multiplier) is $b_0(\cdot)$. But there is a family of polynomials of order 3, which satisfy (16), $b(h) = 0$, and are non-increasing. All of them have the form

$$b(a) = -\frac{9}{4}(c+1)a^3 + \frac{5}{4}(c+1)ha^2 + ch^2a + h^3,$$

where $c \in [-\frac{17}{13}, -\frac{25}{133}]$. The particular choice $c = -1$ gives $b(h - a)$.

convex, then the corresponding qualitative behavior is robust with respect to all sufficiently small twice continuously differential perturbations of $b(a)$.

The above is illustrated in Figure 1, which depicts the result of a numerical exercise based on the following parameter values: $h = 10, b = 10, v(a) = 200, c(a) = 0, s(a) = 18$, and $\tau \in [0, 10]$.

[Figure 1 about here.]

Below we numerically investigate how the deterioration rate and the depreciation rate influence the tax-response of the average age and productivity. Both deterioration and depreciation are aimed to model the impact of aging on the machines, but they take into account different aspects: deterioration reflects the decrease of productivity of the machines, while depreciation reflects the decrease in the number of machines due to break-down. This difference turns out to be significant in the context of emission tax effects.

Consider the same example as above, but with the linear acquisition cost $b(a) = b \cdot (h - a)$, and for three different functions $v(a)$, namely $v(a) = 200$, $v(a) = 200e^{-0.05a}$, and $v(a) = 200e^{-0.1a}$. Figure 2 (left) shows that a higher deterioration rate leads to a lower average age of capital, and to a faster decrease of the average age with τ . The reason is that in this model emissions depend on capital stock and not on production. A higher deterioration rate causes lower productivity of older machines while the emission tax stays the same. Capital stock is kept operational as long as it is profitable, i.e. as long as revenue exceeds the sum of running costs and emission taxes. Revenue decreases faster under a higher deterioration rate so that only younger vintages are kept when the deterioration rate is higher. This effect is magnified if the tax rate is higher since total costs increase with the tax rate.

[Figure 2 about here.]

Now we fix $v(a) = 200$ and let the depreciation rate δ take the three different values $\delta = 0.2, 0.4$, and 0.6 . As before, the higher is the rate, the lower is the average age for zero tax. However, when the tax increases the average age *increases*. The higher is the depreciation rate the faster is the increase of the average age. This can be explained as follows. In contrast with the deterioration rate which only reduces production and thus revenue, the depreciation rate both affects revenue as well as emission taxes negatively. This is because revenue and emission taxes depend on capital stock, and the latter is reduced by depreciation. Under a higher emission tax rate a reduction in the capital stock leads to a larger decrease of emission taxes, which makes it relatively more desirable to produce with older capital goods. Consequently, if depreciation is significant, this "emission tax effect" raises the average age of the capital stock when the emission tax rate is increased. Moreover, this effect is larger for larger depreciation rates.

4 Effects of Learning on Modernization and Productivity

Here we consider the consequences of the occurrence of learning. Learning can appear in two ways: as machines get older it can increase productivity or reduce emissions. Concerning productivity the presence of learning implies that for the function $v(a)$ it holds that $v'(a) > 0$, which reflects that due to learning machines get more productive over the years⁵. It is important to notice

⁵Here it holds that a new machine at time t , say machine 1, has productivity $v(0)$. The same holds for a new machine at time $t + a$, say machine 2. In fact this is the same machine as the one bought a years ago with the only

that modeling learning in this way implies that we implicitly assume perfect spillover of learning. Consider two machines of age a at time t : machine 1 is purchased at time zero while machine 2 is purchased at time $t - \epsilon$. In our model both have the same value of $v(a)$ so that learning also takes place before a machine is purchased. Hence the economy as a whole builds up knowledge about how to work with this machine efficiently, and this knowledge is exchanged freely among firms due to e.g. labor mobility and schooling.

With emissions the presence of learning would result in $s'(a) < 0$, i.e. as machines get older it is learned how to use the machine in a more environmental friendly way. Analogous to productivity learning, also here perfect spillover is implicitly assumed.

First let us check whether this can possibly have an effect on downsizing. To do so we have to differentiate (12) with respect to τ :

$$\frac{\partial K(t, a)}{\partial \tau} = - \left(\int_0^a e^{\int_0^\sigma \delta(\rho) d\rho} \left[\int_\sigma^h e^{-\int_\sigma^\alpha (r + \delta(\rho)) d\rho} s(\alpha) d\alpha \right] d\sigma \right) e^{-\int_0^a \delta(\rho) d\rho} \quad (17)$$

which remains clearly negative without being influenced by changed signs of v' and s' . Hence, also under learning an increase in the emission tax reduces the number of machines of each age in the capital stock, which implies that the firm's total emissions are reduced too.

Now let us turn to the modernization effect and the relation between modernization and productivity. The following proposition summarizes our results.

Proposition 3 *Consider the model set up for the scenario without depreciation and discounting, thus $r = \delta(a) = 0$. If $v(a)$ is not identically zero and non-decreasing in a , $c(a)$ and $s(a)$ are non-increasing and $b(a)$ is convex, then at any time $t \geq h$ it holds that an increase in the emission tax*

i) raises or keeps the optimal average age of the capital stock at the same level. If either $v(a)$ is strictly increasing, or at least one of $c(a)$ and $s(a)$ is strictly decreasing, or $b(a)$ is strictly convex, then the optimal average age strictly increases with τ .

ii) raises or keeps the average productivity of the capital stock at the same level. If $v(a)$ is strictly increasing, then the average productivity strictly increases with τ .

The conclusion of Xepapadeas and De Zeeuw [16] was that when downsizing is accompanied by modernization a stricter environmental policy can increase the average productivity of the capital stock. Our result shows that in the presence of learning this need not be true. In fact, under learning a stricter environmental policy can also result in an increase of average productivity, but now this increase is caused by the presence of older machines. Note that also here the shape of the acquisition cost function $b(a)$ is crucial: in case this function is strictly concave contrary effects arise and no general conclusions can be drawn with respect to the effect of environmental policy on optimal average age and average productivity of the capital stock.

difference that it is of age zero. But the model implies that at time $t + a$ machine 1 will have productivity $v(a)$ which is greater than the productivity $v(0)$ of machine 2. Hence, according to the model the firm should learn again to work on the same machine, which does not make much sense.

It seems that the learning effects like they are modelled here are only present when they are accompanied by technological progress. Still, in this section we choose to model learning separately in order to single out the effects caused by learning. Moreover the results obtained here will also hold in scenarios where technological progress is slow relative to the learning effect.

Profit Effects of Emission Taxes

To analyze the effect of an emission tax on profits, Xepapadeas and De Zeeuw [16] depart from a framework of an international duopoly. The firm in the home country is subject to an emission tax, while the firm in the other country operates without being bothered by environmental regulation. Following Xepapadeas and De Zeeuw [16] we consider a linear demand schedule and we let capital stock be a function of age and the tax rate while leaving out time:

$$p = \bar{p} - \int_0^h v(a) K^\tau(a) da - \int_0^h v(a) K^0(a) da,$$

where we attach the superscript τ to the optimal capital stock to indicate its dependence on τ . It is easy to determine the equilibrium price, which has the form

$$p = p_1\tau + p_0,$$

where

$$p_0 = \frac{\bar{p} + \int_0^h \int_0^a \int_\sigma^h 2v(a) c(\alpha) d\alpha d\sigma da + \int_0^h \int_0^a 2v(a) b(\sigma) d\sigma da}{1 + \int_0^h \int_0^a \int_\sigma^h 2v(a) v(\alpha) d\alpha d\sigma da}$$

and

$$p_1 = \frac{\int_0^h \int_0^a \int_\sigma^h v(a) s(\alpha) d\alpha d\sigma da}{1 + \int_0^h \int_0^a \int_\sigma^h 2v(a) v(\alpha) d\alpha d\sigma da}.$$

The aim is to determine the effect of an emission tax on profits and emissions. To do so, first a benchmark case is defined:

$$\begin{aligned} v(a) &= y, \\ c(a) &= c (= 0), \\ s(a) &= s, \\ b(a) &= b[h - a]. \end{aligned}$$

Then it can be derived that

$$p_0 = \frac{\bar{p} + \frac{2}{3}ybh^3}{1 + \frac{2}{3}h^3y^2}. \quad (18)$$

In what follows we assume that, in the absence of emission taxes, the equilibrium price is sufficiently large so that

$$p_0y - c - b > 0. \quad (19)$$

The benchmark case is compared to a case where productivity varies with age. Xepapadeas and De Zeeuw propose the following specification:

$$\begin{aligned} v(a) &= \kappa y + \frac{8}{3h} [1 - \kappa] y [h - a], \\ c(a) &= c (= 0), \\ s(a) &= s, \\ b(a) &= \left[b + \frac{1}{15} [1 - \kappa]^2 p_0 y \right] [h - a]. \end{aligned} \quad (20)$$

Xepapadeas and De Zeeuw assume that $\kappa \in [0, 1)$, implying that productivity is decreasing in age. In order to introduce productivity learning, implying that productivity increases with age, we just need to assume that $\kappa > 1$. Then the specification under (20) is such that productivity learning is present.

For p_0 it turns out that the specification under (20) leads to the same expression (18) as in the benchmark case. Of course, also for $\kappa > 1$ this expression for p_0 holds.

Next consider p_1 . In the benchmark case we have

$$p_1^b = \frac{\frac{1}{3}ysh^3}{1 + \frac{2}{3}y^2h^3},$$

while in the case of the specification (20) it is obtained that

$$p_1^v = \frac{\frac{1}{3}yh^3s}{1 + \frac{2}{3}\left[1 + \frac{1}{15}[\kappa - 1]^2\right]y^2h^3}. \quad (21)$$

This result holds in case of productivity learning, i.e. $\kappa > 1$, too.

Using this framework the following proposition can be stated for $\kappa \in (1, \kappa^*)$, where $\kappa^* > 1$ is specified in Section 7.3. In this proposition we directly compare the marginal decrease of profit per unit of marginal decrease of emissions, for the case of learning and for the benchmark case.

Proposition 4 *Let $dS^b(\tau)/d\tau$, $dS^v(\tau)/d\tau$, and $d\Pi^b(\tau)/d\tau$, $d\Pi^v(\tau)/d\tau$ denote the marginal decreases in emissions and profits by a stricter environmental policy in the home country, in the benchmark and productivity learning cases, respectively. Then under the assumptions made previously a positive number τ^* can be identified such that for $\tau \in (0, \tau^*)$*

$$\frac{dS^b(\tau)}{d\tau} < \frac{dS^v(\tau)}{d\tau} < 0,$$

and

$$0 < \frac{d\Pi^b}{dS^b}|_{\tau=0} < \frac{d\Pi^v}{dS^v}|_{\tau=0}.$$

The results in Proposition 4 are significantly different from Xepapadeas and De Zeeuw [16]. Their Proposition 3 claims that

$$\left| \frac{dS^v(\tau)}{d\tau} \right| > \left| \frac{dS^b(\tau)}{d\tau} \right|, \quad (22)$$

which in fact means that if productivity decreases due to aging, the reduction in emissions due to stricter environmental policy is larger. If productivity learning is present the reverse is true. This implies that if productivity increases due to learning the reduction in emissions due to stricter environmental policy will be smaller. Thus the presence of productivity learning mitigates the effect of stricter environmental policy with respect to emissions reduction.

Now, we examine the effect of an emission tax on profit. For $\kappa < 1$ Xepapadeas and De Zeeuw obtained the result that

$$\left| \frac{d\Pi^v(\tau)}{d\tau} \right| < \left| \frac{d\Pi^b(\tau)}{d\tau} \right|.$$

This implies that in the case of varying productivity the effect of a stricter environmental policy on profits is lower. However, in Xepapadeas and De Zeeuw only the case of productivity being

decreasing in age was considered. In the case of increasing productivity due to learning we establish that an emission tax that decreases emissions by one unit, leads to a larger decrease of profit, compared to the benchmark case where learning is not present.

The conclusion is that in the presence of productivity learning a stricter environmental policy can have a more negative effect on the economy: to obtain a given emissions reduction the emission tax increase should be higher, and the resulting negative effect on industry profits is larger.

5 Effects of Technological Progress on Modernization and Productivity

To include technological progress we introduce the function $f(t - a)$ (see also Feichtinger et al.[10], and Hartl et al.[11]), where the output of a machine of age a in year t is now given by $v(a) f(t - a)$. We consider the case of process innovation rather than product innovation. This implies that more goods are produced using a capital good of a younger age, but on the other hand that the quality of the produced goods is constant over time. Due to technological progress the output produced by a machine of some fixed age a increases with the vintage $t - a$. More modern machines are more productive because they embody superior technology. Therefore, it holds that $f'(t - a) > 0$.

Due to technological progress a machine will operate more efficiently. Therefore we include the possibility that a machine build at a later date will produce less emissions. To formalize this we impose that the emissions of a machine of age a in year t are given by $f_s(t - a) s(a)$, where $f'_s(t - a) \leq 0$. Similarly we assume that the running costs are $f_c(t - a) c(a)$ with $f'_c(t - a) \leq 0$. It is important to notice that with this model extension Proposition 1 does not hold anymore. This is because in (10) and (12) $v(\alpha)$ needs to be replaced by $f(t - a) v(\alpha)$, which implies that the time invariancy is gone. Hence, the firm does not reach a steady state after h years. Economically, this is easy to understand since the increased productivity over time makes it worthwhile for the firm to keep on growing.

It is easy to see that the downsizing effect of an emission tax arises also here, since also under technological progress the number of machines decreases under a marginal emission tax increase. To study the modernization effect and the relation between modernization and productivity let us specify a simple linear function for $f(t - a)$:

$$f(t - a) = f^0 + \kappa [t - a], \quad (23)$$

with f^0 and κ being nonnegative constants such that $f^0 > \kappa h$. As is shown in Feichtinger et al.[10] a specification like this also covers “Moore’s law” which says that the memory and arithmetic power of micro chips develop exponentially over time, if we assume in addition that production is a logarithmic function of technology.

For the acquisition cost we impose a simple linear relationship:

$$b(a) = b[h - a]. \quad (24)$$

Proposition 5 *Consider the model set up for the scenario with technological progress, where $v'(a) \leq 0$. Assume that the functions f_s and f_c are non-increasing, s is non-decreasing and $\delta(a) = r = 0$. Then at any time $t \geq h$ it holds that an increase in the emissions tax reduces the optimal average age of the capital stock and increases average productivity.*

In fact this is the main result of the Xepapadeas-De Zeeuw paper. We have thus shown that this result carries over to a scenario with technological progress. However, there is one important aspect which comes from the shape of the acquisition cost function. At the end of Section 2 we already mentioned the crucial role of this shape with regard to the optimal average age of the capital stock. That this also holds in the presence of technological progress can be illustrated if we replace the specification in (24) by

$$b_{\gamma}(a) = [b - \gamma a] [h - a],$$

with $\gamma \in [0, 1]$. Note that for $\gamma = 0$ the two functions coincide. We carried out a numerical investigation with specifications $h = 10$, $b = 10$, $v(a) = 200$, $c(a) = 0$, $s(a) = 18$, $\tau = 10$, $f(t - a) = 1 + 0.001[t - a]$, $f_s(t - a) = 1$, for which the outcome is provided in Figure 3.

[Figure 3 about here.]

Again we see the crucial role of the shape of $b(a)$: the intuitively plausible outcome of Proposition 5, i.e. average productivity of the capital stock increases with the emission tax rate, does no longer hold in case the acquisition function $b(a)$ is sufficiently convex. We see that for $\gamma \geq 0.5$ average productivity starts to decrease ($d(\gamma) < 0$) when the emission tax rate increases.

6 Conclusions

This paper belongs to the literature in which the effects of environmental regulation on firm behavior is examined. A specific feature of our framework is that productivity of the firm's machines fluctuates over time. It can increase or decrease with age due to learning and technical deterioration, respectively, and increases with every new vintage due to technological progress.

The contribution that comes most close to our work is Xepapadeas and De Zeeuw [16]. They consider the case of technical deterioration, i.e. productivity of machines decreases with age. For a linear setting they obtain that stricter environmental regulation results in downsizing accompanied by modernization. We found that their results are generalizable to nonlinear functional forms as long as the acquisition costs of investments are concavely decreasing with age, and to the case of technological progress. However, the latter case was only proved for a linear acquisition cost function, and we provided a counterexample in which these acquisition costs were strictly convex. In the presence of learning results are quite different. Now, downsizing is accompanied by increasing the average age of machines, but again this result only holds under specific requirements for the investment acquisition costs. Moreover, where Xepapadeas and De Zeeuw derive that varying productivity due to aging leads to a larger effect of an increased emission tax rate on emission output and a lower effect on profits, in our learning case we obtain a lower effect on emission output. It turns out that an emission tax increase that ensures a given emissions reduction, decreases the profits in the presence of learning more than in the benchmark case where learning does not occur.

Xepapadeas and De Zeeuw conclude that, in connection with stricter environmental policy, "the trade-off between environmental conditions and profits remains but is less sharp because of downsizing and modernization of the industry". We get from our work that this conclusion does not carry over to the learning scenario: in that case the trade off is more likely to be sharper because the emission tax increase should be larger to reach a given emissions output reduction, and the effect on the firm profits is more severe. Hence the conclusion must be that the Porter hypothesis is rejected strongly in the presence of learning effects.

7 Technical Appendix

7.1 Auxiliary Analysis

We start with some preliminary analysis that will be used in the proofs of the propositions.

First we shall extend the formula (12) to the case of technological progress. To do so the functions $v(a)$, $c(a)$ and $s(a)$ in the objective function have to be replaced by $f(t-a)v(a)$, $f_c(t-a)c(a)$, and $f_s(t-a)s(a)$, respectively. As in the case without technological progress, we obtain formally that for $t \geq h$ the optimal evolution of the capital stock is given by

$$K^\tau(t, a) = \int_0^a e^{-\int_\sigma^a \delta(\theta) d\theta} \int_\sigma^h e^{-\int_\sigma^\alpha (\delta(\theta) + r) d\theta} [pf(t-a)v(\alpha) - f_c(t-a)c(\alpha) - \tau f_s(t-a)s(\alpha)] d\alpha d\sigma - \int_0^a e^{-\int_\sigma^a \delta(\theta) d\theta} b(\sigma) d\sigma, \quad (25)$$

provided that $K(t, a)$ given by this formula is nonnegative.

Remark 1 Since we consider an infinite horizon problem, the objective function can be infinite, especially in the case of technological progress or zero discount rate. The optimality should be understood in the following sense (we use the terminology from Carlson, Haurie and Leizarowitz [7]). If there is no technological progress optimality means “decision horizon optimality”, which implies “strong optimality” in case $r > 0$. If there is a technological progress the optimality has the following meaning, rather usual for age-structured control problems: for every $T \geq 0$ there exists $T^* \geq T$ such that for every $T' \geq T^*$ the optimal solution on the finite horizon $[0, T']$ coincides with K^τ (given by (25) on $[0, T]$ (in our case $T^* = T + h$)).

Standing assumptions:

- (i) f , f_c , and f_s are differentiable, strictly positive, and f/f_c and f/f_s are strictly monotone;
- (ii) v , c , and s are not identically zero (except possibly c) monotone⁶, nonnegative, and continuously differentiable;
- (iii) b is twice continuously differentiable, nonnegative and monotone decreasing, $b(h) = 0$; $K_0(a) > 0$ for every $a \in (0, h)$;
- (iv) $K^0(t, a) > 0$ for all t and $a \in [0, h]$.

The above assumptions imply that for a fixed moment T there exists a $\tau_{\max} > 0$ such that $K^\tau(t, a) > 0$ for every $\tau \in [0, \tau_{\max})$, $t \in [0, T]$ and $a \in [0, h]$.

Clearly, for $\tau \in [0, \tau_{\max})$ the optimal capital stock is given by the formula (25). Moreover, the denominator in the definition of $\mathcal{P}_\varphi(\tau)$ is positive.

In the remainder of this section we assume that $\delta(a) = r = 0$.

Then for a fixed $t \geq h$ the optimal capital stock can be written as (here we skip t in the notation, since it is fixed, but indicate the dependence on τ)

$$K^\tau(a) = \int_0^a \left\{ \int_\sigma^h [\hat{f}(a)v(\alpha) - \hat{f}_c(a)c(\alpha) - \tau \hat{f}_s(a)s(\alpha)] d\alpha - b(\sigma) \right\} d\sigma,$$

⁶everywhere in the paper the terms “monotone”, “increasing”, “decreasing” and “convex” are used in their non-strict versions.

where

$$\hat{f}(a) = pf(t-a), \quad \hat{f}_c(a) = f_c(t-a), \quad \hat{f}_s(a) = f_s(t-a),$$

For a given nonnegative strictly monotone function φ we shall investigate how the φ -average

$$\mathcal{P}_\varphi(\tau) = \frac{\int_0^h \varphi(a) K^\tau(a) da}{\int_0^h K^\tau(a) da}$$

is related to the one corresponding to $\tau = 0$.

Below we shall use the symbol \prec for one of the relations \leq or \geq . Moreover, we introduce the notation

$$R(\varphi, q, r) = \int_0^h \int_0^a \int_\sigma^h \varphi(a) q(a) r(\alpha) d\alpha d\sigma da, \quad B(\varphi) = \int_0^h \int_0^a \varphi(a) b(\sigma) d\sigma da.$$

We fix an arbitrary $\tau \in (0, \tau_{\max})$. The inequality $\mathcal{P}_\varphi(\tau) \prec \mathcal{P}_\varphi(0)$ can be written as

$$\frac{R(\varphi, \hat{f}, v) - R(\varphi, \hat{f}_c, c) - \tau R(\varphi, \hat{f}_s, s) - B(\varphi)}{R(1, \hat{f}, v) - R(1, \hat{f}_c, c) - \tau R(1, \hat{f}_s, s) - B(1)} \prec \frac{R(\varphi, \hat{f}, v) - R(\varphi, \hat{f}_c, c) - B(\varphi)}{R(1, \hat{f}, v) - R(1, \hat{f}_c, c) - B(1)}$$

It is easy to verify that the last inequality is implied by the following two inequalities:

$$\frac{R(\varphi, \hat{f}, v)}{R(1, \hat{f}, v)} \prec \frac{R(\varphi, \hat{f}_c, c)}{R(1, \hat{f}_c, c)}, \quad (26)$$

$$\frac{R(\varphi, \hat{f}, v)}{R(1, \hat{f}, v)} \prec \frac{R(\varphi, \hat{f}_s, s)}{R(1, \hat{f}_s, s)}. \quad (27)$$

Therefore it is enough to obtain conditions under which (27) holds. Then the same conditions have to be posed for \hat{f}_c and c instead of \hat{f}_s and s .

Further we assume that $v(0)$ and $s(0)$ are strictly positive. If this is not the case for some of them, say for $v(0)$, then $v(h)$ would be strictly positive since $v(\cdot)$ is monotone and not identically zero. Then in the next integral representation one may use the value $v(h)$ instead of $v(0)$ and finally obtain the same result.

In (27) substitute

$$v(\alpha) = v(0) + \int_0^\alpha v'(\beta) d\beta, \quad s(\alpha) = s(0) + \int_0^\alpha s'(\beta) d\beta.$$

Then it becomes

$$\frac{v(0)R(\varphi, \hat{f}, 1) + V(\varphi) - B(\varphi)}{v(0)R(1, \hat{f}, 1) + V(1) - B(1)} \prec \frac{s(0)R(\varphi, \hat{f}_s, 1) + S(\varphi)}{s(0)R(1, \hat{f}_s, 1) + S(1)}, \quad (28)$$

where

$$V(\varphi) = \int_0^h \int_0^a \int_\sigma^h \int_0^\alpha \varphi(a) \hat{f}(a) v'(\beta)(\beta) d\beta d\alpha d\sigma da,$$

$$S(\varphi) = \int_0^h \int_0^a \int_\sigma^h \int_0^\alpha \varphi(a) \hat{f}_s(a) s'(\beta)(\beta) d\beta d\alpha d\sigma da,$$

Since $v(0)$, $s(0)$ and $R(1, \hat{f}, 1)$ are strictly positive, (28) is implied by the following three ones (skipping further the last argument of R , which is always equal to one):

$$\frac{v(0)R(\varphi, \hat{f}) + V(\varphi) - B(\varphi)}{v(0)R(1, \hat{f}) + V(1) - B(1)} \prec \frac{R(\varphi, \hat{f})}{R(1, \hat{f})} \prec \frac{R(\varphi, \hat{f}_s)}{R(1, \hat{f}_s)} \prec \frac{s(0)R(\varphi, \hat{f}_s) + S(\varphi)}{s(0)R(1, \hat{f}_s) + S(1)}.$$

This implies the following sufficient condition for the relation $\mathcal{P}_\varphi(s(\cdot)) \prec \mathcal{P}_\varphi(0)$:

$$V(\varphi)R(1, \hat{f}) \prec V(1)R(\varphi, \hat{f}) \quad \& \quad B(1)R(\varphi, \hat{f}) \prec B(\varphi)R(1, \hat{f}) \quad (29)$$

$$\& \quad R(\varphi, \hat{f})R(1, \hat{f}_s) \prec R(1, \hat{f})R(\varphi, \hat{f}_s) \quad \& \quad S(1)R(\varphi, \hat{f}_s) \prec S(\varphi)R(1, \hat{f}_s). \quad (30)$$

The first and the last relations are of the same type, but with reversed sign \prec . We separately investigate the first three relations.

1. Changing the order of integration we represent

$$V(\varphi) = \int_0^h G(\varphi; \beta) v'(\beta) d\beta, \quad \text{where} \quad G(\varphi; \beta) = \int_\beta^h \int_0^\alpha \int_\sigma^h \varphi(a) \hat{f}(a) da d\sigma d\alpha.$$

Also by changing the order of integration we obtain that

$$R(\varphi, \hat{f}) = \int_0^h \int_0^\alpha \int_\sigma^h \varphi(a) \hat{f}(a) da d\sigma d\alpha = G(\varphi; 0).$$

Then

$$V(\varphi)R(1, \hat{f}) - W(1)R(\varphi, \hat{f}) = \int_0^h \Delta(\beta) v'(\beta) d\beta, \quad (31)$$

where

$$\Delta(\beta) = G(\varphi; \beta)G(1; 0) - G(1; \beta)G(\varphi; 0).$$

We now prove that $\Delta(\beta)$ does not change its sign in $[0, h]$. Assume the opposite. Clearly $\Delta(0) = 0$ (since $G(\varphi; 0)G(1; 0) - G(1; 0)G(\varphi; 0) = 0$), and $\Delta(h) = 0$ (since $G(1; h) = G(\varphi; h) = 0$). Then $\Delta'(\cdot)$ must have at least two zeros in the open interval $(0, h)$. We calculate

$$\Delta'(\beta) = G'(\varphi; \beta)G(1; 0) - G'(1; \beta)G(\varphi; 0), \quad G'(\varphi; \beta) = - \int_0^\beta \int_\sigma^h \varphi(a) \hat{f}(a) da d\sigma.$$

Since obviously $\Delta'(0) = 0$, we conclude that $\Delta'(\cdot)$ has at least three zeros in $[0, h]$. Then Δ'' must have at least two zeros in the open interval $(0, h)$. We have that

$$\Delta''(\beta) = G''(\varphi; \beta)G(1; 0) - G''(1; \beta)G(\varphi; 0), \quad G''(\varphi; \beta) = - \int_\beta^h \varphi(a) \hat{f}(a) da.$$

Since obviously $\Delta''(h) = 0$, we conclude as before that $G'''(\cdot)$ must have at least two zeros in $(0, h)$. We have that

$$\Delta'''(\beta) = \varphi(\beta) \hat{f}(\beta) G(1; 0) - \hat{f}(\beta) G(\varphi; 0).$$

Since \hat{f} is strictly positive, the equation $\varphi(\beta) G(1; 0) = G(\varphi; 0)$ must have at least two solutions in $(0, h)$. Since φ is a strictly monotone function, we arrive at a contradiction. This contradiction is

caused by the assumption that Δ changes its sign in $[0, h]$. Hence, Δ has a constant sign. To find it we note that $\Delta(0) = \Delta'(0) = 0$, so that

$$\operatorname{sgn} \Delta(\beta) = \operatorname{sgn} \Delta''(0) \quad \forall \beta \in [0, h],$$

provided that $\Delta''(0) \neq 0$. To evaluate $\Delta''(0)$ we represent by changing the order of integration

$$G(\varphi; 0) = \int_0^h \varphi(a) \hat{f}(a) a(h - a/2) da.$$

Then from the expression for $\Delta''(\beta)$ we obtain that

$$\Delta''(0) = \int_0^h \varphi(a) \hat{f}(a) \left[a(h - a/2) \int_0^h \hat{f}(\sigma) d\sigma - \int_0^h \hat{f}(\sigma) \sigma(h - \sigma/2) d\sigma \right] da.$$

Notice that the above expression depends linearly on φ , and equals zero if φ is constant. Then the value does not change if we replace $\varphi(a)$ with $\int_0^a \varphi'(\beta) d\beta$. Changing once again the order of integration we obtain that

$$\Delta''(0) = \int_0^h \varphi'(\beta) \int_\beta^h \int_0^h \hat{f}(a) \hat{f}(\sigma) [a(h - a/2) - \sigma(h - \sigma/2)] d\sigma da d\beta = \int_0^h \varphi'(\beta) \Gamma(\beta) d\beta.$$

Obviously $\Gamma(0) = \Gamma(h) = 0$. If $\Gamma(\beta)$ changes its sign in $[0, h]$, then $\Gamma'(\beta)$ must have at least two zeros in $(0, h)$. We have that

$$\Gamma'(\beta) = -\hat{f}(\beta) \int_0^h \hat{f}(\sigma) [\beta(h - \beta/2) - \sigma(h - \sigma/2)] d\sigma.$$

Since p is strictly positive, the integral must vanish at least two times in $(0, h)$. This, however, cannot happen. Indeed, the integral is a quadratic function of β which takes the same negative value

$$\int_0^h \hat{f}(\sigma) \sigma(h - \sigma/2) d\sigma$$

for $\beta = 0$ and $\beta = 2h$. Hence $\Gamma'(\cdot)$ has at most one zero in $[0, h]$. Since $\Gamma(0) = 0$, we have $\operatorname{sgn} \Gamma(\beta) = \operatorname{sgn} \Gamma'(0)$, if the last value is nonzero. We already showed that $\Gamma'(0)$ is strictly positive. Thus we obtain: if $\varphi'(\beta) \prec 0$ on $[0, t]$, then $\Delta(\beta) \prec 0$. Now we conclude from (31) that:

$$\begin{aligned} & \text{if } 0 \prec v'(\beta) \text{ and } \varphi'(\beta) \prec 0 \text{ on } [0, t], \\ & \text{or } v'(\beta) \prec 0 \text{ and } 0 \prec \varphi'(\beta) \text{ on } [0, t], \\ & \text{then } V(\varphi)R(1, \hat{f}) \prec S(1)R(\varphi, \hat{f}) \text{ is fulfilled.} \end{aligned} \tag{32}$$

Then by the same argument we also have:

$$\begin{aligned} & \text{if } 0 \prec s'(\beta) \text{ and } 0 \prec \varphi'(\beta) \text{ on } [0, t], \\ & \text{or } s'(\beta) \prec 0 \text{ and } \varphi'(\beta) \prec 0 \text{ on } [0, t], \\ & \text{then } S(1)R(\varphi, \hat{f}_s) \prec S(\varphi)R(1, \hat{f}_s) \text{ is fulfilled.} \end{aligned} \tag{33}$$

In both cases, the respective inequalities in (29)–(30) are strict, provided that the two inequalities in the corresponding sufficient condition in (32) or (33) are strict for all $\beta \in [0, h]$.

2. The third relation in (29)–(30) can be rewritten as

$$\int_0^h \varphi(a) \hat{f}(a) a(h-a/2) da - \int_0^h \hat{f}_s(a) a(h-a/2) da - \int_0^h \varphi(a) \hat{f}_s(a) a(h-a/2) da - \int_0^h \hat{f}(a) a(h-a/2) da < 0.$$

Since the above expression is linear in φ and equals zero for a constant function φ , one can replace $\varphi(a)$ with $\int_0^a \varphi'(\beta) d\beta$. After a change of the order of integration the above inequality can be equivalently written as

$$\int_0^h \varphi'(\beta) \Delta(\beta) d\beta < 0, \quad \text{where} \quad \Delta(\beta) = \int_\beta^h a \left(h - \frac{a}{2} \right) \int_0^h \alpha \left(h - \frac{\alpha}{2} \right) [\hat{f}(a) \hat{f}_s(\alpha) - \hat{f}_s(a) \hat{f}(\alpha)] da.$$

We have that

$$\Delta'(\beta) = \beta \left(h - \frac{\beta}{2} \right) \int_0^h \alpha \left(h - \frac{\alpha}{2} \right) [\hat{f}(\beta) \hat{f}_s(\alpha) - \hat{f}_s(\beta) \hat{f}(\alpha)] = \beta \left(h - \frac{\beta}{2} \right) [c_1 \hat{f}(\beta) - c_2 \hat{f}_s(\beta)].$$

Since p/q is assumed strictly monotone, $\Delta'(\beta)$ has at most one zero in $(0, h)$. Since obviously $\Delta(0) = \Delta(h) = 0$, we conclude that $\Delta(\beta)$ does not change its sign in $[0, h]$. Since $\Delta(0) = \Delta'(0) = 0$, one needs to evaluate $\Delta''(0)$ to obtain the sign of $\Delta(\beta)$. We have

$$\Delta''(0) = h[c_1 \hat{f}(0) - c_2 \hat{f}_s(0)] = h \left[\hat{f}(0) \int_0^h \alpha \left(h - \frac{\alpha}{2} \right) \hat{f}_s(\alpha) d\alpha - \hat{f}_s(0) \int_0^h \alpha \left(h - \frac{\alpha}{2} \right) \hat{f}(\alpha) d\alpha \right].$$

According to the standing assumptions $\omega(a) = \hat{f}(a)/\hat{f}_s(a)$ is a strictly monotone function. Moreover

$$\Delta''(0) = h \hat{f}_s(0) \int_0^h \alpha \left(h - \frac{\alpha}{2} \right) \hat{f}_s(\alpha) [\omega(0) - \omega(\alpha)] d\alpha.$$

Since all multipliers of the brackets are strictly positive in $(0, h)$ and the expression within the brackets has a definite sign in this interval, $\Delta''(0)$ has a definite sign as well, and it is determined by the sign of $\omega(0) - \omega(\alpha)$.

Hence we can conclude that

$$\begin{aligned} & \text{if } 0 < \varphi'(a) \text{ and } 0 < (\hat{f}/\hat{f}_s)'(a) \text{ on } [0, h], \\ & \text{or } \varphi'(a) < 0 \text{ and } (\hat{f}/\hat{f}_s)'(a) < 0 \text{ on } [0, h], \\ & \text{then } R(\varphi, \hat{f})R(1, \hat{f}_s) < R(1, \hat{f})R(\varphi, \hat{f}_s) \text{ is fulfilled.} \end{aligned} \quad (34)$$

3. Finally, consider the second relation in (29)–(30), which is

$$B(1)R(\varphi, \hat{f}) - B(\varphi)R(1, \hat{f}) = \quad (35)$$

$$B(1) \int_0^h \varphi(a) \hat{f}(a) a(h-a/2) da - R(1, \hat{f}) \int_0^h \int_0^a \varphi(a) b(\sigma) d\sigma da < 0.$$

Since the above expression is linear in φ and equals zero for a constant φ , we can replace $\varphi(a)$ by $\int_0^a \varphi'(\beta) d\beta$. Changing the order of integration we obtain the equivalent condition

$$\int_0^h \varphi'(\beta) \int_\beta^h \left[B(1) \hat{f}(a) a(h-a/2) - R(1, \hat{f}) \int_0^a b(\sigma) d\sigma \right] da d\beta < 0, \quad (36)$$

where

$$B(1) = \int_0^h (h - \sigma)b(\sigma) d\sigma, \quad R(1, \hat{f}) = \int_0^h \hat{f}(\sigma)\sigma(h - \sigma/2) d\sigma.$$

Thus we obtained that:

$$\begin{aligned} &\text{if (36) holds (or, equivalently, if (35) holds),} \\ &\text{then } B(1)R(\varphi, \hat{f}) \prec B(\varphi)R(1, \hat{f}) \text{ is fulfilled.} \end{aligned} \quad (37)$$

Combining the above considerations we obtain the following auxiliary conclusion.

Lemma 1 *If φ , v , s , f , f_s and b satisfy all conditions listed in (32), (37), (34), (33), and if c and f_c satisfy all those conditions that are required for s and f_s , then $\mathcal{P}_\varphi(\tau) \prec \mathcal{P}_\varphi(0)$.*

7.2 Modernization and Productivity

7.2.1 The case without technological progress

We apply the results from the previous section with $f(t - a) = f_c(t - a) = f_s(t - a) = 1$.

Proof of propositions 2 and 3. We apply Lemma 1 with $\varphi(a) = a$ for the average age, and with $\varphi(a) = v(a)$ for the average productivity. One only needs to elaborate the inequality (36) in these particular cases. Since the multiplier $\varphi'(\beta)$ has a constant sign, we consider the function

$$\Delta(\beta) = \int_\beta^h \left[B(1)a(h - a/2) - R(1, 1) \int_0^a b(\sigma) d\sigma \right] da,$$

where $R(1, 1) = h^3/3$. Obviously $\Delta(h) = 0$, and taking into account the definition of $B(1)$ we get that also $b(0) = 0$. If $\Delta(\beta)$ changes its sign in $[0, h]$ then $\Delta'(\beta)$ must have at least two zeros in $(0, h)$. But since

$$\Delta'(\beta) = B(1)\beta(h - \beta/2) - R(1, 1) \int_0^\beta b(\sigma) d\sigma,$$

which has one more zero at $\beta = 0$, we conclude that $\Delta''(\beta)$ must have at least two zeros in $(0, h)$. But

$$\Delta''(\beta) = B(1)(h - \beta) - R(1, 1)b(\beta),$$

which has an additional zero at $\beta = h$. Hence, also $\Delta'''(\beta)$ must have two zeros in $(0, t)$. This means that the equation

$$B(1) + R(1, 1)b'(\beta) = 0$$

has at least two zeros. Since the function $b'(\cdot)$ is assumed monotone, this is possible only if it is a constant, which means that $b(a) = b'(0)(b - h)$. One can directly check that in this case $\Delta(\beta)$ is identically zero, therefore it does not change its sign as well.

Knowing that $\Delta(\beta)$ does not change its sign, and also that $\Delta(0) = \Delta'(0)$ we obtain that

$$\text{sgn } \Delta(\beta) = \text{sgn } \Delta'(0),$$

provided that $\Delta'(0) \neq 0$. We have that

$$\Delta'(0) = hB(1) + R(1, 1)b(0).$$

Let us represent

$$b(\sigma) = \frac{b(0)}{h}(h - \sigma) + \delta(\sigma),$$

where obviously $\delta(0) = \delta(h) = 0$. Then

$$\Delta'(0) = -h \int_0^h (h - \sigma) \delta(\sigma) d\sigma.$$

We see that if $\delta(\sigma)$ has a constant sign, then $\Delta(\beta)$ has the opposite sign. It remains to notice that $\delta(\sigma)$ is positive for a concave function $b(\cdot)$ and negative for a convex one. We conclude that

$$\begin{aligned} & \text{if } 0 \prec \varphi'(a) \text{ and } b''(a) \prec 0 \text{ on } [0, h], \\ & \text{or } \varphi'(a) \prec 0 \text{ and } 0 \prec b''(a) \text{ on } [0, h], \\ & \text{then } B(1, \hat{f})R(\varphi, 1) \prec B(\varphi)R(1, 1) \text{ is fulfilled.} \end{aligned} \quad (38)$$

Then Lemma 1 implies propositions 2 and 3.

Q.E.D.

7.2.2 The case with technological progress

Proof of Proposition 5. We apply Lemma 1 with

$$\hat{f}(a) = f_0(t - a) = f^0 + \kappa \cdot (t - a), \quad b(a) = b \cdot (h - a),$$

As before, we only need to elaborate condition (36), which can be written as

$$\int_0^h \dot{\varphi}(\beta) \Delta(\beta) d\beta \leq 0, \quad (39)$$

where

$$\Delta(\beta) = \int_\beta^h [B(1)\hat{f}(a)a(h - a/2) - R(1, \hat{f}) \int_0^a b(\sigma) d\sigma] da.$$

One can calculate that $\Delta(\beta) = -b\kappa \frac{h^3}{48} \beta^2 (h - \beta)(5h - 2\beta)$, which is negative in $(0, h)$. Then the proposition follows from Lemma 1. Q.E.D.

7.3 Profit Effects

Proof of Proposition 4. The validity of the considerations below is established for values of the parameter κ in some interval $\kappa_* < 1 < \kappa^*$ and values $\tau \in (0, \tau^*]$, $\tau^* > 0$. For such values of κ and τ the capital stock at the optimal solution is supposed positive, so that the formulas from Section 2 are valid. The value $\kappa^* > 1$ will be a subject to some more constraints specified below.

First consider the emissions result. It will be convenient to introduce the following notation:

$$\begin{aligned} d_0(\kappa) &= 1 + \frac{1}{3}\left[1 + \frac{1}{15}(1 - \kappa^2)\right]y^2h^3, \\ d_1(\kappa) &= 1 + \frac{1}{3}\left[1 + \frac{2}{15}(1 - \kappa^2)\right]y^2h^3, \\ d_2(\kappa) &= 1 + \frac{2}{3}\left[1 + \frac{1}{15}(1 - \kappa)^2\right]y^2h^3, \end{aligned}$$

and to suppose that κ^* is chosen in such a way that $d_i(\kappa) > 0$ for $\kappa \in [1, \kappa^*]$, which is always possible.

In Xepapadeas and De Zeeuw [16] it is obtained that

$$\frac{dS^b(\tau)}{d\tau} = -\frac{s^2h^3}{3} \frac{d_1(1)}{d_2(1)}, \quad \frac{dS^v(\tau)}{d\tau} = -\frac{s^2h^3}{3} \frac{d_1(\kappa)}{d_2(\kappa)}.$$

We suppose that $\kappa^* > 1$ is chosen in such a way that for $0 \leq \kappa < \kappa^*$ the numerator in the last ratio is positive. Then

$$\frac{dS^b(\tau)}{d\tau} < \frac{dS^v(\tau)}{d\tau}$$

is equivalent to each of the next inequalities obtained successively:

$$\frac{s^2h^3}{3} \frac{d_1(\kappa)}{d_2(\kappa)} < \frac{s^2h^3}{3} \frac{d_1(1)}{d_2(1)}, \quad (40)$$

$$5(1 - \kappa^2) < 4(1 - \kappa)^2,$$

$$-9\kappa^2 + 8\kappa + 1 < 0,$$

and the last inequality is true for $\kappa > 1$. This proves the first claim of the proposition.

Next we investigate the effect of emission tax on profits. For the benchmark case Xepapadeas and De Zeeuw [16] obtained that

$$\frac{d\Pi^b(\tau)}{d\tau} = \frac{1}{3}sh^3 \left[\left(\frac{d_1(1)}{d_2(1)} \right)^2 s\tau - \frac{d_1(1)}{d_2(1)}(p_0y - c - d) \right]$$

while for the case of varying productivity they derived that

$$\frac{d\Pi^v(\tau)}{d\tau} = \hat{\pi}\tau + \tilde{\pi},$$

where

$$\hat{\pi} = \frac{1}{45} [1 - \kappa]^2 (p_1^v)^2 y^2h^3 + \frac{s^2h^3}{3} \left[\frac{d_1(\kappa)}{d_2(\kappa)} \right]^2,$$

and

$$\tilde{\pi} = -\frac{sh^3}{3d_2(\kappa)} \left[d_0(\kappa)p_0y - d_1(\kappa)[c + b + \frac{1}{15}(1 - \kappa)^2p_0y] \right].$$

Rearranging the terms one obtains that

$$\tilde{\pi} = -\frac{sh^3}{3d_2(\kappa)} \left[d_1(\kappa)(p_0y - c - d) - \frac{1}{45}(1 - \kappa^2)y^2h^3p_0y - \frac{1}{15}d_1(\kappa)(1 - \kappa)^2p_0y \right].$$

Then (19) implies that $\tilde{\pi} < 0$ if $\kappa^* > 1$ is chosen sufficiently close to one.

From the above formulas we get that

$$\begin{aligned}
0 < \frac{d\Pi^v}{dS^v}|_{\tau=0} &= \frac{\frac{d\Pi^v}{d\tau}(0)}{\frac{dS^v}{d\tau}(0)} = \frac{\frac{sh^3}{3d_2(\kappa)} [d_1(\kappa)(p_0y - c - d) - \frac{1}{45}(1 - \kappa^2)y^2h^3p_0y - \frac{1}{15}d_1(\kappa)(1 - \kappa)^2p_0y]}{\frac{s^2h^3}{3} \frac{d_1(\kappa)}{d_2(\kappa)}} \\
&= \frac{p_0y - c - b}{s} - \frac{3d_2(\kappa)}{s^2h^3d_1(\kappa)} \frac{1}{15}(1 - \kappa)p_0y \left[\frac{1}{3}(1 + \kappa)y^2h^3 + d_1(\kappa)(1 - \kappa) \right] \\
&= \frac{d\Pi^b}{dS^b}|_{\tau=0} + (\kappa - 1)r(\kappa).
\end{aligned}$$

Obviously the remainder $r(\kappa)$ is positive if $\kappa^* > 1$ is chosen sufficiently close to one. Hence, for $\kappa \in (1, \kappa^*]$ we obtain the last claim of the proposition.

Q.E.D.

References

- [1] Barucci, E., Gozzi, F., 2001, Technology adoption and accumulation in a vintage capital model, *Journal of Economics*, 52, 159-188.
- [2] Benhabib, J., Rustichini, A., 1991, Vintage capital, investment, and growth, *Journal of Economic Theory*, 55, 323-339.
- [3] Boucekkine, R., Licandro, O., Christopher, P., 1997, Differential-difference equations in economics: on the numerical solution of vintage capital growth models, *Jopurnal of Economic Dynamcis and Control*, 21, 347-362.
- [4] Boucekkine, R., Germain, M., Licandro, O., Magnus, A., 1998, Creative destruction, investment volatility, and the average age of capital, *Journal of Economic Growth*, 3, 361-384.
- [5] Boucekkine, R., del Rio, F., Licandro, O., 1999, Endogenous vs exogenously driven fluctuations in vintage capital models, *Journal of Economic Theory*, 88, 161-187.
- [6] Boucekkine, R., Germain, M., Licandro, O., Magnus, A., 2001, Numerical solution by iterative methods of a class of vintage capital models, *Journal of Economic Dynamics and Control*, 25, 655-669.
- [7] Carlson, D., Haurie, A., Leizarowitz, A., 1991, *Infinite Horizon Optimal Control*, Springer-Verlag, Berlin.
- [8] Feichtinger, G., Hartl, R.F., 1986, *Optimale Kontrolle Oekonomischer Prozesse: Anwendungen des Maximumprinzips in den Wirtschaftswissenschaften*, de Gruyter, Berlin.
- [9] Feichtinger, G., Hartl, R.F., Kort, P.M., Veliov, V.M., 2001, Dynamic investment behavior taking into account aging of the capital good, *CentER Discussion Paper No. 2001-13*, Tilburg University.

- [10] Feichtinger, G., Hartl, R.F., Kort, P.M., Veliov, V.M., 2002, Anticipation effects of technological progress on capital accumulation: a vintage capital approach, Working Paper 272.
- [11] Hartl, R.F., Kort, P.M., Veliov, V.M., Feichtinger, G., 2001, Capital accumulation under technological progress and learning: a vintage capital approach, Working Paper 271.
- [12] Malcomson, J.M., 1975, Replacement and the rental value of capital equipment subject to obsolescence, *Journal of Economic Theory*, 10, 24-41.
- [13] Porter, M., 1991, America's green strategy, *Scientific American*, 264, 96
- [14] Porter, M., Linde, C. van der, 1995, Toward a new conception of the environment-competitiveness relationship, *Journal of Economic Perspectives*, 9, 97-118.
- [15] Solow, R., Tobin, J., Von Weizacker, C.C., Yaari, M., 1966, Neoclassical growth with fixed-factor productions, *Review of Economic Studies*, 33, 79-115.
- [16] Xepapadeas, A., De Zeeuw, A., 1999, Environmental policy and competitiveness: the Porter hypothesis and the composition of capital, *Journal of Environmental Economics and Management*, 37, 165-182.

List of Figures

1	The functions b_0 , b_1 and b_2 (left) and the corresponding average age of capital, if $s = 18$ with $\tau \in [0, 10]$ (right).	25
2	Average age for different deterioration rates (left) and different depreciation rates (right).	26
3	Representatives of the family of functions $b_\gamma(a)$ for different values of γ (left); the difference, $d(\gamma)$, in the average productivity for $\tau = 10$ and $\tau = 0$, as a function of γ (right).	27

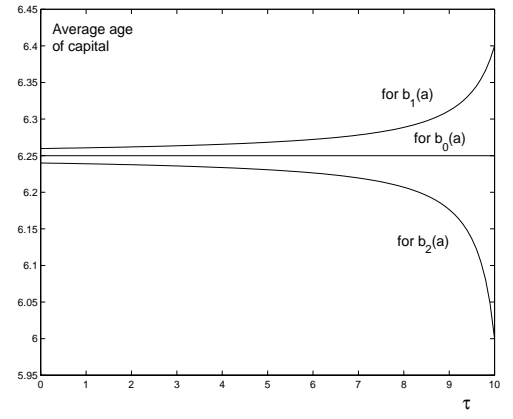
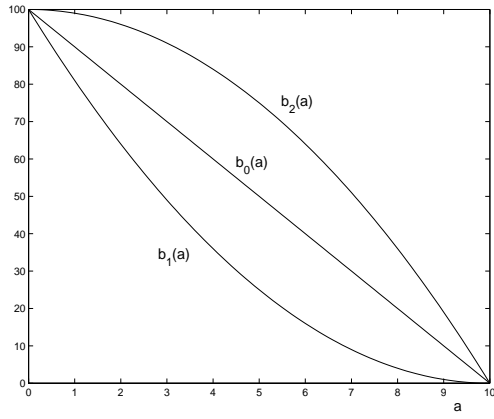


Figure 1: The functions b_0 , b_1 and b_2 (left) and the corresponding average age of capital, if $s = 18$ with $\tau \in [0, 10]$ (right).

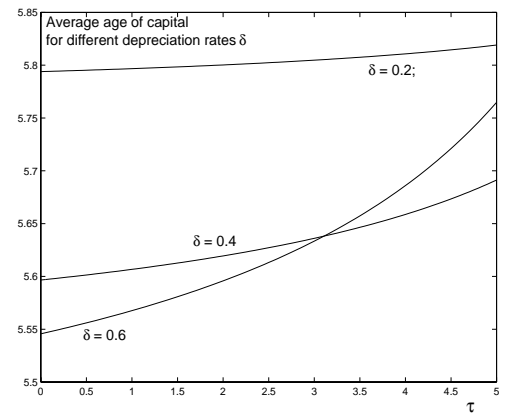
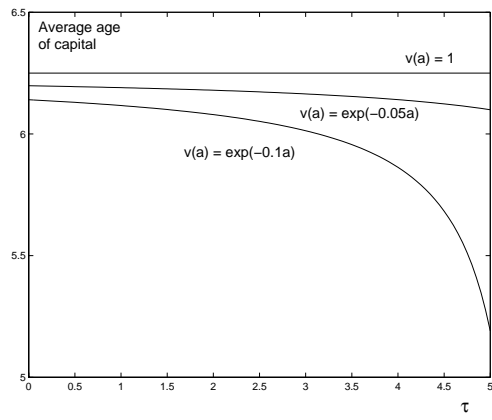


Figure 2: Average age for different deterioration rates (left) and different depreciation rates (right).

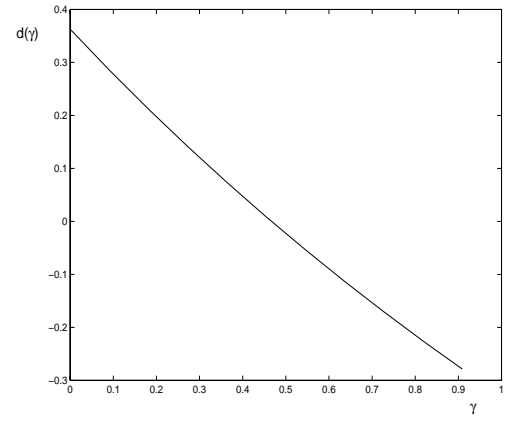
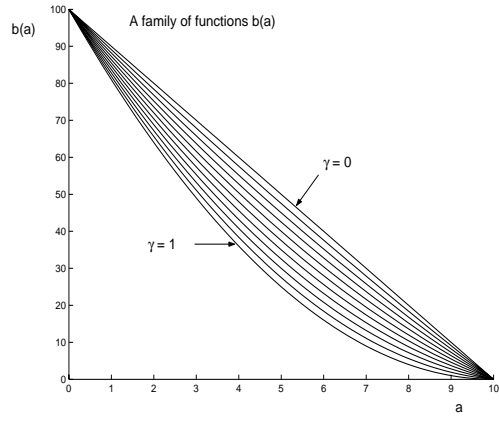


Figure 3: Representatives of the family of functions $b_\gamma(a)$ for different values of γ (left); the difference, $d(\gamma)$, in the average productivity for $\tau = 10$ and $\tau = 0$, as a function of γ (right).